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ANALYTICAL METHOD FOR DETERMINING TRANSMISSION AND ABSORPTION OF TIME-DEPENDENT RADIATION THROUGH THICK ABSORBERS

II - SOURCE INTENSITY, TIME-DEPENDENT

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ANALYTICAL METHOD FOR DETERMINING TRANSMISSION AND ABSORPTION OF TIME-DEPENDENT RADIATION THROUGH THICK ABSORBERS

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SUMMARY

Matrix methods are employed in the solution of absorption problems in which the incident radiation is a known arbitrary function of time. The case of a plane source of polychromatic radiation of several types at normal incidence to a plane absorber is considered.

The method is applied to a hypothetical example in which the amounts of radiations transmitted and reflected as well as the amounts converted to thermal energy are found for a time-dependent source.

INTRODUCTION

A mathematical analysis of the interaction of nuclear radiations with matter has been applied to the case where radioactivity of the absorber is time-dependent (reference 1). The intensities of sources of nuclear radiations, however, may also be time-dependent. For example, U^{235}, an alpha emitter with a half-life of 7.1×10^8 years decays exponentially to Th^{231}, a beta emitter with a half-life of 24.6 hours that in turn becomes Pa^{231}, another alpha emitter with a half-life of 3.2×10^4 years. By the time the decay process is complete, the final product being stable Pb^{207}, the original U^{235} has become a mixture of 13 isotopes of 11 elements and the nuclear radiations that are emitted from this mixture consist of alpha rays, beta rays, and gamma rays of various energies and intensities. Thus, an absorber designed to protect personnel from the effects of the nuclear radiations emitted by U^{235} would be inadequate unless it could adequately diminish all these radiations and also the radiations due to radioactivity induced in the absorber by all the radiations.

Conversely, Cu^{58}, a positron emitter with a half-life of 81 seconds, decays exponentially to stable Ni^{58} and consequently would require no shielding whatever after 1 day – an obvious result that is not predicted in a theory that assumes constancy of source intensity.

In order to enable shielding calculations of such time-dependent sources to be made, an extension of the methods of reference 1 to include the case in which the incident radiation is a known arbitrary function of time was developed at the NACA Lewis laboratory and is presented. Explicit use of the more general case, in which the radiation absorbed may consist of any finite number of types, is shown herein.

SYMBOLS

For the most part, the definitions of all the symbols used agree with the definitions in reference 1. Brackets around the matrix terms have, however, been omitted whenever no decrease in clarity results. The following symbols are used in this report:

$[A],[B],[C],[D]$	matrix coefficients
$F(\tau)$	matrix whose elements are known functions of time
H	matrix of thermal power generated
h	matrix of rate of conversion of energy of nuclear radiation absorbed to thermal energy
I_i	matrix of power of radiation from $(i-1)^{th}$ station incident on i^{th} station
n	number of stations in absorber
P	matrix whose elements are one-half of power of radiation from radioactivity
R_i	matrix of power of radiation from i^{th} station incident on $(i-1)^{th}$ station
r	matrix of power back-scattering coefficient

S_i	i^{th} station of absorber
t	matrix of power-transmission coefficient
λ	matrix of decay constant for radioactivity, $second^{-1}$
μ_i	matrix of rate of conversion of energy of radiation absorbed in S_i to energy of radioactivity in S_i
τ	time, seconds

Subcripts:

0	initial condition
i	i^{th} station of absorber
τ	time

Superscripts:

$a,b,\ldots j,\ldots q$	types of radiation
n	neutron radiation
γ	gamma radiation

ANALYSIS

The analysis is based on the following assumptions:

(1) Elements of absorber are parallel plane surfaces.

(2) Incident radiation is a known arbitrary function of time.

(3) Incident radiation consists of several types of polychromatic radiation.

(4) Radiation is normal to absorber.

(5) Part of radiation absorbed at each station of absorber is transformed to thermal energy and part to induced radioactivity in the absorber.

(6) Radioactive atoms in the absorber decay in a single step to stable atoms.

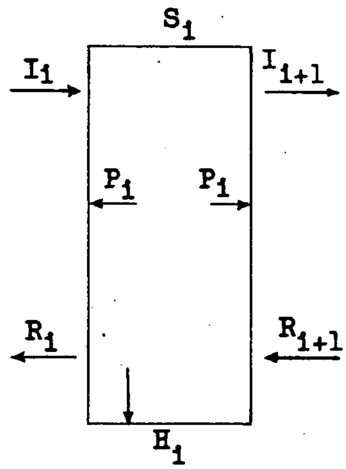
(7) Products of radioactive decay within the absorber may consist of types and energies that are the same or different from those of the incident radiation.

(8) One-half of the radioactivity produced in each station of absorber is emitted from each side of the absorber.

(9) Radiation impinging upon source R_1 has a negligible effect on the source intensity.

Method. - Diagram (a) is included to clarify the discussion in the following paragraphs.

The absorber may be considered as consisting of n stations normal to the path of radiation. The i^{th} station of the absorber is denoted by S_i. Radiation I_i from the preceding station S_{i-1} is incident upon one surface of S_i, whereas radiation R_{i+1} from the subsequent element S_{i+1} impinges upon the opposite surface of S_i. From S_i emerges radiation I_{i+1}, which is incident upon S_{i+1}, and radiation R_i, which is incident upon S_{i-1}.



(a)

In the treatment presented herein, the following quantities are assumed to be known:

$I_1(\tau)$ radiation power incident upon absorber from source

$R_{n+1}(\tau)$ radiation power incident upon absorber from side of absorber opposite source

$(P_i)_0$ one-half of initial power of radioactivity in i^{th} station of absorber

$(H_i)_0$ initial thermal power being emitted from i^{th} station of absorber h matrix of rate of conversion of energy of nuclear radiation absorbed to thermal energy

r matrix of power back-scattering coefficient

t matrix of power-transmission coefficiant

λ matrix of radioactive decay constant

μ matrix of rate of conversion of energy of radiation absorbed to energy of radioactivity

All of the previous quantities except the first two are time-independent constants.

The problem is to find I_i, R_i, and H_i in terms of known quantities. The defining equations and conditions for q types of radiation are:

$$\left.\begin{aligned}
I_{i+1}^{a} &= t_i^{a} I_i^{a} + r_i^{a} R_{i+1}^{a} + P_i^{a} \\
R_i^{a} &= r_i^{a} I_i^{a} + t_i^{a} R_{i+1}^{a} + P_i^{a} \\
\frac{dP_i^{a}}{d\tau} &= \mu_i^{a,a} (I_i^{a} + R_{i+1}^{a}) + \ldots + \mu_i^{j,a} (I_i^{j} + R_{i+1}^{j}) + \ldots + \\
&\quad \mu_i^{q,a} (I_i^{q} + R_{i+1}^{q}) - \lambda_i^{a} P_i^{a} \\
&\vdots \\
I_{i+1}^{j} &= t_i^{j} I_i^{j} + r_i^{j} R_{i+1}^{j} + P_i^{j} \\
R_i^{j} &= r_i^{j} I_i^{j} + t_i^{j} R_{i+1}^{j} + P_i^{j} \\
\frac{dP_i^{j}}{d\tau} &= \mu_i^{a,j} (I_i^{a} + R_{i+1}^{a}) + \ldots + \mu_i^{j,j}(I_i^{j} + R_{i+1}^{j}) + \ldots + \\
&\quad \mu_i^{q,j} (I_i^{q} + R_{i+1}^{q}) - \lambda_i^{j} P_i^{j} \\
&\vdots
\end{aligned}\right\} \qquad (1)$$

$$
\left.\begin{aligned}
I_{i+1}^{q} &= t_i^{q} I_i^{q} + r_i^{q} R_{i+1}^{q} + P_i^{q} \\
R_i^{q} &= r_i^{q} I_i^{q} + t_i^{q} R_{i+1}^{q} + P_i^{q} \\
\frac{dP_i^{q}}{d\tau} &= \mu_i^{a,q} \left(I_i^{a} + R_{i+1}^{a}\right) + \ldots + \mu_i^{j,q} \left(I_i^{j} + R_{i+1}^{j}\right) + \ldots + \\
&\qquad \mu_i^{q,q} \left(I_i^{q} + R_{i+1}^{q}\right) - \lambda_i^{q} P_i^{q} \\
H_i &= h_i^{a} \left(I_i^{a} + R_{i+1}^{a}\right) + \ldots + h_i^{j} \left(I_i^{j} + R_{i+1}^{j}\right) + \ldots + h_i^{q} \left(I_i^{q} + R_{i+1}^{q}\right)
\end{aligned}\right\} \quad (1)\ \text{Continued}
$$

where the double superscripts signify a conversion of radiation of the type represented by the first superscript to the type represented by the second superscript.

Equations (1) are used to find P. As in Case III of reference 1, all the I's and R's are expressed in terms of $I_1(\tau)$ and $R_{n+1}(\tau)$, which are known functions of time and of P's. The results may be expressed in the form

$$
\frac{d[P]}{d\tau} = [A][P] + [B]\left[I_1(\tau)\right] + [C]\left[R_{n+1}(\tau)\right] + [D] \tag{2}
$$

where $[A]$, $[B]$, $[C]$, and $[D]$ are matrices, all the elements of which are constants.

Let

$$
[B]\left[I_1(\tau)\right] + [C]\left[R_{n+1}(\tau)\right] + [D] = \left[F(\tau)\right]
$$

Then if $[I_1(\tau)]$ and $[R_{n+1}(\tau)]$ are independent of $[P]$, the solution to equation (2) may be written as

$$
[P] = e^{[A]\tau}\left\{[P]_0 + \int e^{-[A]\tau}\left[F(\tau)\right] d\tau\right\} \tag{3}
$$

Thus, $[P]$ is reduced to a quadrature. Substitution of the expressions for $\left[I_1(\tau)\right]$ and $\left[R_{n+1}(\tau)\right]$ into equation (3) results in an explicit expression for P. After P is known, I_i, R_i, and H_i may be found.

Previous knowledge of all the nuclear reactions that will occur during the time interval in which the calculations are made is necessary; for if even one energy or type of radiation is present but not accounted for, all the calculations may be invalidated.

Example. - Assume that:

(1) The absorber is two stations in thickness.

(2) Energy is not degraded nor upgraded.

(3) The incident radiation is 1.00 Mev gamma radiation.

(4) The incident gamma radiation induces a radioactivity of 2 Mev neutrons in the second station of the absorber.

(5) $I_1 = \begin{bmatrix} I_1^{\gamma} \\ I_1^{n} \end{bmatrix} = \begin{bmatrix} (1 - \sin \omega\tau) \\ 0 \end{bmatrix}$ roentgen per hour

$\omega = \pi \times 10^{-3}$ second^{-1}

$R_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ roentgen per hour

$t_1^{\gamma} = 0.5$

$t_2^{\gamma} = 0.8$

$t_1^{n} = 0.9$

$t_2^{n} = 0.9$

$r_1^{\gamma} = 0.1$

$r_2^{\gamma} = 0.05$

$r_1^{n} = 0.1$

$r_2^{n} = 0.1$

$$\mu_1^{\gamma,\gamma} = \mu_2^{\gamma,\gamma} = \mu_1^{\gamma,n} = \mu_1^{n,n} = \mu_1^{n,\gamma} = \mu_2^{n,\gamma} = \mu_2^{n,n} = 0$$

$$\mu_2^{\gamma,n} = 10^{-6} \text{ second}^{-1}$$

$$\lambda_2^{n} = 10^{-5} \text{ second}^{-1}$$

$$h_1^{\gamma} = h_1^{n} = h_2^{n} = 0$$

$$h_2^{\gamma} = 10^{-3}$$

$$\left[P_1^{\gamma}\right]_0 = \left[P_2^{\gamma}\right]_0 = \left[P_1^{n}\right]_0 = 0$$

At $\tau = 0$, $P_2^{n} = 0.1$ roentgen per hour. The problem is to determine the values of I_2, I_3, R_1, R_2, and H_2 at $\tau = 10^5$ seconds.

<u>Solution</u>. - If equations (1) are applied to the previous data and brackets are omitted for simplicity, then

$$I_2^{\gamma} = 0.5\ I_1^{\gamma} + 0.1\ R_2^{\gamma}$$

$$I_3^{\gamma} = 0.8\ I_2^{\gamma}$$

$$R_1^{\gamma} = 0.1\ I_1^{\gamma} + 0.5\ R_2^{\gamma}$$

$$R_2^{\gamma} = 0.05\ I_2^{\gamma}$$

or in terms of I_1^{γ},

$$I_2^{\gamma} = 0.503\ I_1^{\gamma}$$

$$I_3^{\gamma} = 0.402\ I_1^{\gamma}$$

$$R_1^{\gamma} = 0.113\ I_1^{\gamma}$$

$$R_2^{\gamma} = 0.025\ I_1^{\gamma}$$

Also

$$I_2^{n} = 0.1\ R_2^{n}$$

$$I_3^{n} = 0.9\ I_2^{n} + P_2^{n}$$

$$R_1^{n} = 0.9\ R_2^{n}$$

$$R_2^{n} = 0.1\ I_2^{n} + P_2^{n}$$

from which

$$I_2^n = 0.101\ P_2^n$$

$$I_3^n = 1.091\ P_2^n$$

$$R_1^n = 0.909\ P_2^n,$$

$$R_2^n = 1.010\ P_2^n$$

Now

$$\frac{dP_2^n}{d\tau} = 10^{-6}\ \text{second}^{-1} \times 0.503\ I_1^\gamma - 10^{-5}\ \text{second}^{-1}\ P_2^n$$

Or

$$P_2^n = e^{-10^{-5}\tau}\left\{0.1 + \int 5.03 \times 10^{-7}\ \ e^{-10^{-5}\tau} I_1^\gamma(\tau)\ d\tau\right\}$$

$$= e^{-10^{-5}\tau}\left\{0.1 + 5.03 \times 10^{-7} \int e^{10^{-5}\tau}(1 - \sin\omega\tau)\ d\tau\right\}$$

$$= \frac{e^{-10^{-5}\tau}}{10} + 5.03 \times 10^{-7}\left[\frac{1}{10^{-5}} + \frac{\omega\cos\omega\tau - 10^{-5}\sin\omega\tau}{\omega^2 + 10^{-10}}\right]$$

At $\tau = 10^5$ seconds,

$$P_2^n = 0.0873\ \text{roentgen per hour}$$

Therefore

$$I_2^\gamma = 0.503\ \text{roentgen per hour}$$

$$I_3^\gamma = 0.402\ \text{roentgen per hour}$$

$$R_1^\gamma = 0.113\ \text{roentgen per hour}$$

$$R_2^\gamma = 0.025\ \text{roentgen per hour}$$

$$I_2^n = 0.0088\ \text{roentgen per hour}$$

$$I_3{}^n = 0.0952 \text{ roentgen per hour}$$

$$R_1{}^n = 0.0794 \text{ roentgen per hour}$$

$$R_2{}^n = 0.0882 \text{ roentgen per hour}$$

$$H_2 = 10^{-3} I_2{}^{\gamma} = 5.03 \times 10^{-4} \text{ roentgen per hour}$$

DISCUSSION

In practice, most sources of radioactivity are time-dependent. This time-dependency varies from the simple exponential natural decay law obeyed by all radioactive substances to the sharply varying functions of dynamic piles. Thus, there are many cases in which the amount of shielding material may safely be less than that required to diminish adequately the maximum instantaneous output of a source. Application of the method described herein provides a frequently quick answer to the amount of shielding needed for a given source.

In general, the right side of equation (3) includes an integral of an arbitrary function. In such cases, approximate and numerical methods of integration may be applied to obtain explicit values for P_i. This method requires no further integration and all values of I_i, R_i, and H_i can be found by elementary algebraic manipulations involving P_i.

The main limitations of the method are a lack of sufficient experimental data on the absorption of products of radioactivity and on the rates of conversion of these radiations to secondary radiations and to thermal energy as in reference (1). Consideration of the geometry must also be made; a plane source at normal incidence to a plane absorber is assumed throughout. The magnitude of the effects of deviations from this ideal case must be known for reliable results in a given set of calculations.

SUMMARY OF RESULTS

Matrix methods are employed in the solution of absorber problems in which the incident radiation is an arbitrary known function of time. The solution is obtained in the form of a quadrature.

The method presented is applied to a hypothetical example in which the incident radiation consists of a source of gamma radiation that varies sinusoidally with the time.

Lewis Flight Propulsion Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, May 12, 1949.

REFERENCE

1. Allen, G.: Analytical Method for Determining Transmission and Absorption of Time-Dependent Radiation through Thick Absorbers. I - Radioactivity of Absorber, Time-Dependent. NACA TN 1919, 1949.